**Quad-rotor dynamics report**

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# Introduction

In this paper I will describe the dynamical system of Quad-Rotor (aka quad), with its full 6 Degree-of-Freedom (orientation/attitude and position). Later on I will add also a slung load, which will increase the system’s number of D.O.F.

I will describe the derivation of the equations of motion. And show some limiting cases to verify the model.

The afterall purpose is to analyze the system stability acording to ‘weakly-nonlinear’ method.

# Numenclature

σ : solidity ratio

a : 2D lift coefficient slope

μ : advance ratio

λ : inflow ratio

𝛎 : induced velocity

ρ : air density

: rotor radius

: pitch of incidence

: twist pitch

: drag coefficient at 70% radial station

l : distance propeller axis to CoG

h : distance propeller plane to CoG

: rotation matrix from Body to Inertial coordinate system

: rotation matrix from Inertial to Body coordinate system

# The system model

General

The quad-rotor has 4 body arms and 4 electircal motors – each produces thrust dependent on the rotation speed of the rotors blades. At nominal state, all the rotors are rotating at the same constant speed, with direction shown in Fig.1. The engines are numbered 1-4 as shown in the figure.



Figure 1

Rotors 1 and 3 are both Clockwise, rotors 2 and 4 are both CounterClockwise.

## Dimensional dynamical state

I will describe the system in 3 sections :

1. Acting forces and moments
2. Kinematics of the quad
3. Energies and equations of motion

**Assumptions** for the model :

1. Ground effect is *not* taken in account because the desired balance point for investigation is above ground. (usually G.E is considered when at least away)

1.a disturbance on the payload, as result of downwash is also not taken in account. Possibly we will in the future.

1. The quad body and parts are assumed to be rigid. *No* elastisity is considered.
2. Body geometry and mass is (almost) symetrical. Hence the Inertia matrix is pure diagonal. (this issue might need to be reconsidered in later phase of the work)

1. **Forces** acting on the rotors and the body frame :

The External forces are derived from aerodynamic reactions - creating forces and moments on the quad body(frame) and the motors themselfes and hence the body hub. by a combination of momentum and blade element theory (as in helicopters).

All the next mentioned forces and moments are in BODY framework.

1. Thrust of *each* rotor (i = 1..4) , acts perpendicular to the blades plane:
2. Side force on each of the motors’ Hubs. it is the outcome of the forces on the blades in their horizontal direction.

The resultant **moments** on the system are:

1. Drag moment - the result of moements by drag forces on the blades ( in their horizontal direction )
2. Rolling moment - integration over the entire rotor of the lift of each section acting at a given radius

2. **Kinematics** , and rotation matrices

I will consider the general coordinates of the quad:

is the location in space. Relative to inertial coor. system.

is the orientation relative to the inertial coor. system.

Hence the 6 D-O-F.

Description of the rotaion between vector in the body coor.system to description in the inretial frame-work:

I will use the ‘Tait–Bryan angles’ notion.

rotation about axis in Clockwise direction (negative right hand direction)

rotation about Y’ axis in Clockwise direction (negative right hand direction)

(Y’ is the intermidiate Y-axis of the ‘temporary’ coordinate system of euler angles)

rotation about axis in Clockwise direction (negative right hand direction)

and so the overall rotation matrix (by matrix multiplication) from Body to Inertial is :

( the other-way rotation matrix will be )

When given Body coordinates one can represent them in Inertial system by:

Angular velocity vector of the quad is :

For describing that vector in Body-coor.system we need to describe it in the relevant axes :

=

=

Define =

Using the 1st vector definition above we get , in body coordinates system , the (relative) angular rate :

= =

another thing the is influenced by the rotated coor.system is Moment of Inertia.

The matrix I = is in Body system.

The inertia in inertial system is : (rotated about the axis of rotation).

3. Energies of the system :

In order to derive the equations of motion, I’ll use the Lagraunge method.

Therefor I’ll use the lagrangian which is :

*Kinetic* energy ( translational + rotational contributioins ) :

*Potential* energy:

(z positive is downwards)

Position of vector in Body system is : location of the C.G with addition of location in that system:

* V=

As long as the system is only a rigid body then v= , and (the angular rates we showed above).

T=

(again – it would be more complicated argument if we considered a non symetric rigid body)

The Lagrange equation are:

,

,

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Some of the used trigonometric identities are:

the general forces are :

– drag of the body frame and

- drag on the body frame parts, that are below the rotors and affected by the rotors downwash.

Reffs:

1. S. BOUABDALLAH , “design and control of quadrotors with application to autonomous flying”, THÈSE NO 3727 (2007), Lausanne, EPFL